

SUPERCURRENT NOISE IN SNS JUNCTIONS

D.V. AVERIN

Department of Physics and Astronomy, SUNY at Stony Brook, Stony Brook NY 11794

Quasiclassical Green's functions of a short superconductor/normal metal/superconductor (SNS) junction are calculated for multi-mode junction with arbitrary electron transmission properties. They provide the basis for description of all of the junction transport characteristics, and are used to calculate the equilibrium spectral density of supercurrent fluctuations.

1 Introduction

The process of multiple Andreev reflections (MAR) is the dominant mechanism of electron transport in Josephson junctions with large electron transparency. One of the main qualitative features of MAR is the coherent transfer of charge by large quanta, the size of which increases with decreasing bias voltage V . At low bias voltages, $eV \ll \Delta$, where Δ is the superconducting energy gap, this leads to strong increase of non-equilibrium current noise in the MAR regime, the fact that presently attracts interest to the noise properties of high-transparency junction both in theory^{1,2,3} and experiment^{4,5,6}. At V smaller than the quasiparticle energy relaxation rate γ , $eV \ll \hbar\gamma$, the MAR-noise saturates and turns into equilibrium supercurrent noise that is best described in terms of the thermal fluctuations of the occupation factors of the Andreev states carrying supercurrent. Since the dynamics of these occupation factors is driven by inelastic transitions, comprehensive theory of the current noise in high-transparency junctions should be based on the Green's functions formalism which provides a rigorous description of the inelastic processes. Until now, such a theory was developed only for the single-mode ballistic junctions¹. The aim of this work is to calculate quasiclassical Green's functions of a short multi-mode Josephson junction with arbitrary electron transmission properties. They are used then to derive the general expression for equilibrium supercurrent noise and to find this noise in disordered SNS junctions with diffusive electron transport.

2 Junction model and Green's functions

The basic model of a short Josephson junction with large electron transparency is a constriction between two superconductors that supports N propagating electron modes. If the constriction length d is much smaller than the superconductor coherence length ξ , the only characteristic of the constriction relevant for electron transport is the elastic scattering matrix S :

$$S = \begin{pmatrix} r & t \\ t' & r' \end{pmatrix}, \quad (1)$$

which is energy-independent in the relevant energy range set by the superconducting energy gap. Unitarity of S implies that the $N \times N$ transmission and reflection matrices can be expressed as: $t = u_1^\dagger \sqrt{D} u_2$, $t' = u_3^\dagger \sqrt{D} u_4$, $r = u_1^\dagger \sqrt{R} u_4$, $r' = -u_3^\dagger \sqrt{R} u_2$, where D is the diagonal matrix of N transmission probabilities, $R = 1 - D$, and u_j are unitary matrices.

Transport characteristics of such junctions can be expressed in terms of their quasiclassical Green's functions. Calculation of these functions for the multi-mode junctions presented here follows closely similar calculation⁷ in the single-mode case. The quasiclassical Green's functions can be written as

$$G(z, z') = \sum_{\pm} (E_{\pm} e^{\pm i k_F(z-z')} + F_{\pm} e^{\pm i k_F(z+z')}), \quad (2)$$

where z, z' are the coordinates along the current flow, and k_F is the Fermi momentum. The amplitudes E and F are slowly varying functions of coordinates in the junction electrodes and have matrix structure in the transverse mode space. Because of the electron scattering, they change rapidly in the junction region ($z \simeq 0$). Since the junction is short, $d \ll \xi$, rapid variation of the amplitudes E and F across it can be described as the boundary condition relating them at $z \rightarrow -0$ and $z \rightarrow +0$. This condition is derived from the fact that in the junction region, $G(z, z')$ satisfy regular Schrödinger equation as a function of z and z' , and therefore can be expressed in terms of the products of the two scattering states,

$$G(z, z') = \sum_{i,j=1,2} \psi_i(z) \lambda_{ij} \psi_j(z'). \quad (3)$$

Here ψ_1 is the solution of the Schrödinger equation for particles incident from the left,

$$\psi_i(z) = \begin{cases} e^{ik_F z} + r e^{-ik_F z}, & z \rightarrow -\infty, \\ t e^{ik_F z}, & z \rightarrow \infty, \end{cases} \quad (4)$$

and ψ_2 is similar solution for scattering from the right. Note that “infinity” in eq. (4) means $|z| \gg d$ and still corresponds to $z \simeq 0$, i.e. $|z| \ll \xi$, on the length scale of variations of the amplitudes E and F in the electrodes that is given by the coherence length ξ . Comparison of expansion of $G(z, z')$ obtained from eqs. (3), (4), and similar equation for ψ_2 , in the region to the left of the junction with the expansion to the right establishes four relations between the amplitudes E and F in these two regions:

$$\begin{aligned} t^\dagger (F_+(+0) - E_+(+0)r') &= (F_+(-0) - r^\dagger E_-(-0))t, \\ (F_-(-0) - r'^\dagger E_+(-0))t' &= t^\dagger (F_-(-0) - E_-(-0)r), \\ t^\dagger E_-(-0)t &= E_-(-0) + r'^\dagger E_+(-0)r' - F_-(-0)r' - r'^\dagger F_+(-0), \\ t'^\dagger E_+(-0)t' &= E_+(-0) + r^\dagger E_-(-0)r - F_+(-0)r - r^\dagger F_-(-0). \end{aligned} \quad (5)$$

“Rotating” the amplitudes E and F by the unitary matrices u_j we can bring all coefficients in eqs. (5) into the diagonal form in the transverse mode space. The equilibrium Green’s functions $G_{1,2}$ deep inside first and second superconductors, which play the role of the source terms in equations defining the amplitudes, are proportional to unit matrix in this space. This means that the boundary conditions (5) become diagonal for the appropriately rotated amplitudes. In terms of the combinations of these amplitudes

$$\begin{aligned} Q_1^\pm &= (u_4 F_+(-0) u_1^\dagger \pm u_1 F_-(-0) u_4^\dagger)/2, & Q_2^\pm &= (u_3 F_+(+0) u_2^\dagger \pm u_2 F_-(-0) u_3^\dagger)/2, \\ P_1^- &= (u_4 E_+(-0) u_4^\dagger - u_1 E_-(-0) u_1^\dagger)/2, & P_1^+ &= (u_4 E_+(-0) u_4^\dagger + u_1 E_-(-0) u_1^\dagger)/2 - G_1, \\ P_2^- &= (u_3 E_+(+0) u_3^\dagger - u_2 E_-(-0) u_2^\dagger)/2, & P_2^+ &= (u_3 E_+(+0) u_3^\dagger + u_2 E_-(-0) u_2^\dagger)/2 - G_2, \end{aligned}$$

the boundary conditions (5) can be written as follows:

$$\begin{aligned} P_1^- &= P_2^-, & Q_1^- &= Q_2^-, \\ Q_1^+ - Q_2^+ &= \sqrt{R}(P_1^+ + P_2^+ + G_1 + G_2), \\ G_1 - G_2 + P_1^+ - P_2^+ &= \sqrt{R}(Q_1^+ + Q_2^+). \end{aligned} \quad (6)$$

Requirement that all deviations of the Green’s functions from the equilibrium homogeneous values $G_{1,2}$, that are represented by the non-vanishing amplitudes P and Q , decay away from the junction region (at $z \rightarrow \pm\infty$) imposes another conditions on P and Q . This condition has the same form as in the single-mode case⁷:

$$G_j P_j^\pm = -P_j^\pm G_j = (-1)^{j+1} P_j^\mp, \quad G_j Q_j^\pm = Q_j^\pm G_j = (-1)^{j+1} Q_j^\mp, \quad j = 1, 2. \quad (7)$$

Combined, eqs. (6) and (7) give a system of linear matrix equations for the amplitudes P and Q . Solving this system and using the normalization condition satisfied by the Green's functions $G_{1,2}$, $G_{1,2}^2 = 1$,⁹ we get finally:

$$\begin{aligned} E_+ &= u_4^\dagger D(G_1 + 1)G_- G_+ / K u_4 + G_1, & F_+ &= u_4^\dagger \sqrt{R}(G_1 + 1) / K u_1, \\ E_- &= u_1^\dagger D(G_1 - 1)G_- G_+ / K u_1 + G_1, & F_- &= u_1^\dagger \sqrt{R}(G_1 - 1) / K u_4, \end{aligned} \quad (8)$$

where $G_\pm = (G_1 \pm G_2)/2$ and $K = 1 - DG_-^2$. The amplitudes E and F in eq. (8) are given in the first electrode ($z \rightarrow -0$). Similar expressions can be obtained for the second electrode.

Equations (8) solve the problem of electron transport in different types of junctions ranging from quantum point contacts with few propagating electron modes to short multi-mode SNS junctions with diffusive electron transport. They are valid for arbitrary ratio of the two values of the superconducting energy gap in the junctions electrodes, and can be applied, e.g., to symmetric SNS junctions with the same gap and NS junctions with vanishing gap in one of the electrodes. The amplitudes (8) define the Green's functions (2) in the junction region and allow one to calculate different transport characteristics of the junction. For example, since the single-mode version of eqs. (8) was the starting point of the theory⁸ of ac Josephson effect in high-transparency junctions, eqs. (8) prove that this theory can be extended automatically to the multi-mode junctions. Another application of eqs. (8) is presented in the next Section, where they are used to calculate equilibrium fluctuations of the supercurrent.

3 Equilibrium supercurrent noise

General expression for the current noise in terms of the quasiclassical Green's functions is derived in¹⁰. In equilibrium, this equation reduces to the following result for the spectral density of current fluctuations¹:

$$S_I(\omega) = \frac{e^2}{32\pi^2\hbar} \sum_{\pm\omega, \pm} \int d\epsilon f(\epsilon)(1 - f(\epsilon \pm \omega)) \text{Tr}[\rho_\pm(\epsilon)\sigma_z\rho_\pm(\epsilon \pm \omega)\sigma_z - \nu_\pm(\epsilon)\sigma_z\nu_\mp(\epsilon \pm \omega)\sigma_z], \quad (9)$$

where σ_z is the Pauli matrix, the trace is taken over the transverse mode space and over electron-hole space, $\rho(\epsilon) \equiv E^R(\epsilon) - E^A(\epsilon)$, and $\nu(\epsilon) \equiv F^R(\epsilon) - F^A(\epsilon)$. The functions with superscripts R, A denote retarded and advanced components of the Green's functions.

In what follows, the noise (9) is calculated for a symmetric junction between two identical superconductors which are close to "ideal" BCS superconductors:

$$G_1^{R,A}(\epsilon) = \frac{1}{((\epsilon \pm i\gamma_1)^2 + (\Delta \pm i\gamma_2)^2)} \begin{pmatrix} \epsilon \pm i\gamma_1 & (\Delta \pm i\gamma_2)e^{i\varphi/2} \\ -(\Delta \pm i\gamma_2)e^{-i\varphi/2} & -(\epsilon \pm i\gamma_1) \end{pmatrix}. \quad (10)$$

Here φ is the Josephson phase difference across the junction. Imaginary parts $\gamma_{1,2}$ of ϵ , Δ arise from inelastic scattering, for instance the electron-phonon scattering, and are assumed to be small, $\gamma_{1,2} \ll \epsilon, \Delta$. The functions $G_2^{R,A}$ in the second electrode are given by the same expression with $\varphi \rightarrow -\varphi$. For superconductors described by the Green's functions (10), the amplitudes (8) contain resonant denominators $1 - DG_-^2$, the contribution from which dominates the noise at low frequencies $\omega \ll \Delta/\hbar$. Indeed, in this case $G_-^2 = \tilde{\Delta}^2 \sin(\varphi/2)/(\tilde{\Delta}^2 - \tilde{\epsilon}^2)$, where $\tilde{\Delta} = \Delta \pm i\gamma_2$ and $\tilde{\epsilon} = \epsilon \pm i\gamma_1$, and $1 - DG_-^2$ nearly vanishes for $\epsilon \simeq \epsilon_k \equiv \pm\Delta[1 - D_k \sin^2(\varphi/2)]^{1/2}$. Then,

$$\rho(\epsilon), \nu(\epsilon) \propto \sum_{\pm} \frac{\gamma_k/2}{(\epsilon \mp \epsilon_k)^2 + \gamma_k^2/4}, \quad (11)$$

where $\gamma_k = 2(\gamma_1(\epsilon_k) \mp \gamma_2(\epsilon_k)\epsilon_k/\Delta)$. (Since $\gamma_1(-\epsilon) = \gamma_1(\epsilon)$ and $\gamma_2(-\epsilon) = -\gamma_2(\epsilon)$, the width γ of the resonance is the same for both resonances in eq. (11).) The two resonances correspond to the

two discrete Andreev states per mode with energies $\pm\varepsilon_k$ in the subgap region. Their broadening γ is caused by the inelastic transitions between these states and the continuum of states in the bulk electrodes. Expression for γ in the case of electron-phonon transitions is given, e.g., in¹¹.

Due to resonance (11), we can take all non-resonant terms out of the integral over ε in eq. (9). Evaluating the non-resonant terms at $\varepsilon = \pm\varepsilon_k$ and intergrating the resonant denominators we obtain the spectral density of current fluctuations at low frequencies $\omega \sim \gamma$:

$$S_I(\omega) = \frac{1}{2\pi} \sum_{k=1}^N \left(\frac{I_k}{\cosh(\varepsilon_k/2T)} \right)^2 \frac{\gamma_k}{\omega^2 + \gamma_k^2}, \quad (12)$$

where $I_k = (e\Delta^2/2\hbar)D_k \sin \varphi/\varepsilon_k$ is the contribution to the supercurrent from one of the Andreev states in the k th mode.

Equation (12) determines the equilibrium noise of the supercurrent in short Josephson junctions. It has a simple interpretation in terms of the fluctuations of the occupation factors of Andreev states carrying the supercurrent. All states are occupied independently one from another. If a state with energy $\pm\varepsilon_k$ is occupied, it gives contribution $\pm I_k$ to the supercurrent; if it is empty, the contribution to the supercurrent is zero. These two situations are realized with the probabilities $f(\pm\varepsilon_k)$ and $1 - f(\pm\varepsilon_k)$, respectively, and the transitions between them occur with the characteristic rate γ . Calculation of the spectral density of this simple classical stochastic process reproduces eq. (12). This equation was conjectured in¹ on the basis of the similar interpretation of the noise in the single-mode ballistic junctions. The calculation presented in this work provides its rigorous proof.

At temperatures close to the superconducting critical temperature T_c of the junction electrodes, the energy gap is small, $T \gg \Delta$, and the transition rates γ_k and the occupation probabilities of Andreev states are independent of the state energy, and eq. (12) can be simplified further. In this high-temperature regime, one also has to take into account the non-resonant contribution to the current noise from the continuum part of the junction spectrum which can become comparable to the resonant contribution (12). As can be seen from eq. (9), this non-resonant contribution coincides at $T \gg \Delta$ with the regular equilibrium thermal noise of a normal junction. Combining the two contributions we obtain the noise in the high-temperature limit:

$$S_I(\omega) = \frac{1}{2\pi} \left(\sum_{k=1}^N I_k^2 \right) \frac{\gamma}{\omega^2 + \gamma^2} + S_N, \quad (13)$$

where $S_N = GT/\pi$ and $G = (e^2/\pi\hbar) \sum_{k=1}^N D_k$ is the junction conductance.

Equation (13) can be evaluated if the junction has known distribution of transparencies. An important example is provided by the regular multi-mode disordered SNS junction with diffusive electron transport. This type of junctions is characterized by the quasicontinuous Dorokhov's distribution of transparencies, $\sum_k = (\pi\hbar G/2e^2) \int_0^1 dD/D(1-D)^{1/2}$. Calculating the integral over D we find spectral density of current noise at large temperatures:

$$S_I(\omega) = \frac{G\Delta^2}{2\hbar} \cos^2 \frac{\varphi}{2} \left(\frac{\varphi}{\sin \varphi} - 1 \right) \frac{\gamma}{\omega^2 + \gamma^2} + S_N, \quad \varphi \in [-\pi, \pi]. \quad (14)$$

The phase dependence of the supercurrent-noise part of eq. (14) is plotted in Fig. 1. Equation (14) shows that if $T \ll \Delta^2/\hbar\gamma$, this noise gives the dominant contribution to the current noise at frequencies $\omega \sim \gamma$. The noise can also be larger than the average supercurrent $I(\varphi) = (\pi G\Delta^2/4eT) \sin \varphi$. If $\Delta/T < (e^2/\hbar G)^{1/2}$, typical r.m.s. of the supercurrent noise (in the frequency range γ) is larger than $I(\varphi)$. This means that the supercurrent noise should qualitatively change the behavior of the SNS junctions at temperatures close to T_c and should also be observable at low temperatures.

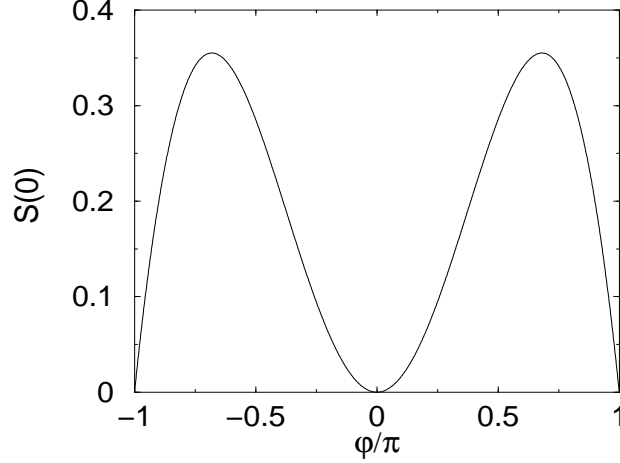


Figure 1: Dependence of the zero-frequency spectral density $S(0) = (S_I(0) - S_N)/(G\Delta^2/2\hbar\gamma)$ of the supercurrent noise in short disordered SNS junctions on the Josephson phase difference φ at large temperatures $T \gg \Delta$. All quantities are defined as in the eq. (14).

4 Conclusion

In conclusion, the equilibrium supercurrent noise has been studied in short Josephson junctions with arbitrary electron transmission properties. Characteristic qualitative features of the noise are its dependence on the Josephson phase difference across the junction, and the cutoff frequency determined by the relaxation rate, which can be much smaller than the temperature and superconducting energy gap. The r.m.s. of the noise is larger than the average supercurrent at temperatures close to the critical temperature of the junction electrodes. A formal result of this work is the general expression for the quasiclassical Green's functions of the multi-mode junctions that can be used to study other transport characteristics, for instance, the non-equilibrium noise in the MAR regime.

Acknowledgments

This work was supported in part by ONR grant # N00014-95-1-0762 and by the U.S. Civilian Research and Development Foundation under Award No. RP1-165.

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